

gr-qc/9912056 v2

## STATIONARY AND NONSTATIONARY SCALAR VACUUM FIELD NOISES

HARET C. ROSU \*

*Instituto de Física, Universidad de Guanajuato, Apdo Postal E-143, León, Gto, Mexico*

**Summary.**- If stationary, the spectrum of vacuum field noise (VFN) is an important ingredient to get information about the curvature invariants of classical worldlines (relativistic classical trajectories). For scalar quantum field vacua there are six stationary cases as shown by Letaw some time ago, these are reviewed here. However, the non-stationary vacuum noises are not out of reach and can be processed by a few mathematical methods which I briefly comment on. Since the information about the kinematical curvature invariants of the worldlines is of radiometric origin, hints are given on a more useful application to radiation and beam radiometric standards at relativistic energies.

## 1. Introduction

Soon after Hawking's theoretical discovery of black hole evaporation, a number of authors have used scalar relativistic quantum field theory in order to show that apparently there exist thermal radiation effects due entirely to vacuum fluctuations in the case of relativistic noninertial motion. In particular, Unruh [1] studied a uniformly linearly accelerated two-level detector of DeWitt (DW) monopole type moving in flat space and proved that for it the Minkowski vacuum looks like a heat bath at a temperature  $T_U = a/2\pi$  ( $\hbar = c = 1$ ), where  $a$  is the proper acceleration. Although not a realistic result, it is simple and general, and connected to Hawking's temperature that may be considered the case in which  $a$  is the gravitational acceleration at the Schwarzschild horizon. By 1986, Takagi wrote an excellent review where he used the concept of vacuum noise for the excitations of the vacuum as seen in noninertial relativistic motion [2]. However, for more general trajectories two or all three kinematical invariants of Frenet type get involved, not just the curvature one as in the case of linear acceleration. A more detailed study of scalar vacuum noise, with significant results has been accomplished by Letaw, either alone or in collaboration with Pfautsch [3]. My aims here are (i) to review Letaw's results, that I present in a different perspective in which I emphasize the radiometric features of the curvature invariants of stationary worldlines and (ii) to provide a few hints on the methods that may be used to analyze the nonstationary cases.

## 2. Six types of stationary scalar VFNS

In general, the scalar quantum field vacua do not possess stationary vacuum excitation spectra (abbreviated as SVES) for all types of classical relativistic trajectories

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\*e-mail: rosu@ifug3.ugto.mx

on which the DW detector moves. Nevertheless, the linear uniform acceleration is *not* the only case with that property as was shown by Letaw who extended Unruh's considerations, obtaining six types of worldlines with SVES for DW monopole detectors (SVES-1 to SVES-6, see below). The line of arguments is the following. The DW detector is effectively immersed in a scalar bath. Its rate of excitation is determined by the energy spectrum of the scalar bath that can be expressed as the density of states times a cosine Fourier transform of the Wightman correlation function (WCF) of the scalar field. Since the WCF is directly expressed in terms of the inverse of the geodetic interval what one needs to calculate is a Fourier transform of the inverse of the geodetic interval  $dx_\mu^2$ . Some calculations are sketched in the Appendix. For stationarity, one should assume the WCF as dependent only on the proper time interval. As shown by Letaw, the stationary worldlines are solutions of some generalized Frenet equations on which the condition of constant curvature invariants is imposed, i.e., constant curvature  $\kappa$ , torsion  $\tau$ , and hypertorsion  $\nu$ , respectively. Notice that one can employ other frames such as the Newman-Penrose spinor formalism as recently did Unruh [4] but the Frenet-Serret one is in overwhelming use throughout physics. The six stationary cases are the following

1.  $\kappa = \tau = \nu = 0$ , (inertial, uncurved worldlines). SVES-1 is a trivial cubic spectrum

$$S_1(E) = \frac{E^3}{4\pi^2} , \quad (1)$$

i.e., as given by a vacuum of zero point energy per mode  $E/2$  and density of states  $E^2/2\pi^2$ .

2.  $\kappa \neq 0, \tau = \nu = 0$ , (hyperbolic worldlines). SVES-2 is Planckian allowing the interpretation of  $\kappa/2\pi$  as 'thermodynamic' temperature. In the dimensionless variable  $\epsilon_\kappa = E/\kappa$  the vacuum spectrum reads

$$S_2(\epsilon_\kappa) = \frac{\epsilon_\kappa^3}{2\pi^2(e^{2\pi\epsilon_\kappa} - 1)} . \quad (2)$$

3.  $|\kappa| < |\tau|, \nu = 0, \rho^2 = \tau^2 - \kappa^2$ , (helical worldlines). SVES-3 is an analytic function corresponding to the case 4 below only in the limit  $\kappa \gg \rho$

$$S_3(\epsilon_\rho) \xrightarrow{\kappa/\rho \rightarrow \infty} S_4(\epsilon_\kappa) . \quad (3)$$

Letaw plotted the numerical integral  $S_3(\epsilon_\rho)$ , where  $\epsilon_\rho = E/\rho$  for various values of  $\kappa/\rho$ .

4.  $\kappa = \tau, \nu = 0$ , (the spatially projected worldlines are the semicubical parabolas  $y = \frac{\sqrt{2}}{3}\kappa x^{3/2}$  containing a cusp where the direction of motion is reversed). SVES-4 is analytic, and since there are two equal curvature invariants one can use the dimensionless energy variable  $\epsilon_\kappa$

$$S_4(\epsilon_\kappa) = \frac{\epsilon_\kappa^2}{8\pi^2\sqrt{3}} e^{-2\sqrt{3}\epsilon_\kappa} . \quad (4)$$

It is worth noting that  $S_4$ , being a monomial times an exponential, is rather close to the Wien-type spectrum  $S_W \propto \epsilon^3 e^{-\text{const.}\epsilon}$ .

5.  $|\kappa| > |\tau|$ ,  $\nu = 0$ ,  $\sigma^2 = \kappa^2 - \tau^2$ , (the spatially projected worldlines are catenaries, i.e., curves of the type  $x = \kappa \cosh(y/\tau)$ ). In general, SVES-5 cannot be found analytically. It is an intermediate case, which for  $\tau/\sigma \rightarrow 0$  tends to SVES-2, whereas for  $\tau/\sigma \rightarrow \infty$  tends toward SVES-4

$$S_2(\epsilon_\kappa) \xrightarrow{0 \leftarrow \tau/\sigma} S_5(\epsilon_\sigma) \xrightarrow{\tau/\sigma \rightarrow \infty} S_4(\epsilon_\kappa) . \quad (5)$$

6.  $\nu \neq 0$ , (rotating worldlines uniformly accelerated normal to their plane of rotation). According to Letaw, SVES-6 forms a two-parameter set of curves. These trajectories are a superposition of the constant linearly accelerated motion and uniform circular motion. The corresponding vacuum spectra have not been calculated by Letaw, not even numerically.

Thus, only the hyperbolic worldlines, having just one nonzero curvature invariant, allow for a Planckian SVES and for a strictly one-to-one mapping between the curvature invariant  $\kappa$  and the ‘thermodynamic’ temperature ( $T_U = \kappa/2\pi$ ). The VFN of semicubical parabolas can be fitted by Wien-type spectra, the radiometric parameter corresponding to both curvature and torsion. The other stationary cases, being nonanalytical, lead to approximate determination of the curvature invariants defining locally the classical worldline on which a relativistic quantum particle moves.

### 3. Preferred vacua and/or high energy radiometric standards

There is much interest in considering the magnetobremssstrahlung radiation patterns at accelerators in the aforementioned perspective [5]. The ‘thermal quantum field vacuum’ standpoint has been initiated by the works of Bell and collaborators during 1984-1987 [6]. In this sense, a sufficiently general and acceptable statement on the *universal* nature of the kinematical Frenet parameters occurring in a few important quantum field model problems can be formulated as follows

*There exist accelerating classical trajectories (worldlines) on which moving ideal (two-level) quantum systems can detect the scalar vacuum environment as a stationary quantum field vacuum noise with a spectrum directly related to the curvature invariants of the worldline, thus allowing for a radiometric meaning of those invariants.*

Although this may look an extremely ideal (unrealistic) formulation for accelerator radiometry [7], where the spectral photon flux formula of Schwinger,<sup>8</sup> is very effective (synchrotron flux  $\propto \xi \int_{\xi}^{\infty} K_{5/3}(z) dz$ , with the variable  $\xi$  scaled in terms of the radian frequency at maximum power), I recall that Hacyan and Sarmiento [9] developed a formalism similar to the scalar case to calculate the vacuum stress-energy tensor of the electromagnetic field in an arbitrarily moving frame and applied it to a system in uniform rotation, providing formulas for the energy density, Poynting flux and stress of zero-point oscillations in such a frame. Moreover, Mane [10] has suggested the Poynting flux of Hacyan and Sarmiento to be in fact synchrotron radiation when it is coupled to an electron.

Another important byproduct, that I put forth in a previous work [11], is the possibility to choose a class of preferred vacua of the quantum world [12] as *all* those having stationary vacuum noises with respect to the classical (geometric) worldlines of *constant* curvature invariants because in this case one may find some necessary attributes of universality in the more general quantum field radiometric sense [13], in which the Planckian Unruh thermal spectrum is included as a particularly important case. Of course, much work remains to be done for a more “experimental”

(i.e., realistic) picture of highly academic calculations in quantum field theory, but a careful look to the literature shows that there are already definite steps in this direction [14]. Even Unruh, in one of his last studies on the ‘orbiting electron’, had to conclude that it does respond to a ‘thermal’ bath, but one with a frequency-dependent temperature [15]. One should notice that the heat-bath picture and the associated VFNs look extremely ideal from the experimental standpoint and not very convincing with respect with the more definite and well-settled quantum electrodynamical calculations. Indeed, it is known that only strong external fields can make the quantum electrodynamical vacuum to react and show its physical properties, becoming similar to a magnetized and polarized medium, and only by such means one can learn about the nature of the instabilities and the physical structure of the QED vacuum. For important results regarding the relationship between Schwinger mechanism and Unruh effect, the reader is directed to some recent works [16]. I also would like to point out that the VFN stochastic processes are directly related to the motion of single particles that would be difficult to disentangle within the bunches of particles circulating in storage rings. The bunch motion is the result of more common and powerful sources able to produce various stochasticity features of the beam [17]. There were proposals to detect VFNs in somewhat cleaner environments, such as Penning traps of *geonium* type, in which the motion of a single electron can be monitored for a long time [18].

#### 4. Nonstationary VFNs

The nonstationary VFNs have a time-dependent spectral content requiring joint time and frequency information, i.e. generalizations of the power spectrum analysis such as tomographical processing [19] and wavelet transform analysis [21]. The so-called non-commutative tomography (NCT) transform  $M(s, \mu, \nu)$  proposed by Manko and Vilela Mendes seems to be an attractive way of processing the analytic nonstationary signals and perhaps one can hope to get unambiguous information for nonanalytic signals as well. In the definition of  $M$ ,  $s$  is just an arbitrary curve in the non-commutative time-frequency plane. The most simple examples have been given by Manko and Vilela Mendes by means of the axes  $s = \mu t + \nu \omega$ , where  $\mu$  and  $\nu$  are linear combination parameters. The NCT transform is related to the Wigner-Ville quasidistribution  $W(t, \omega)$  by an invertible transformation of the form

$$M(s, \mu, \nu) = \int \exp[-ik(s - \mu t - \nu \omega)] W(t, \omega) \frac{dk d\omega dt}{(2\pi)^2}. \quad (6)$$

According to Man’ko and Vilela Mendes the well-known interpretation ambiguities of the Wigner-Ville transform can be avoided in the case of NCT transform, i.e., a rigorous probability interpretation can be assigned. Other useful properties are  $M(t, 1, 0) = |f(t)|^2$  and  $M(\omega, 0, 1) = |f(\omega)|^2$ , where  $f$  is the analytic signal which is simulated by  $M$ . But perhaps the most interesting property of  $M$  is the possibility to detect the presence of signals in noises for small signal-to-noise ratios. This last property may be very effective for detecting VFNs which are very small ‘signals’ with respect to more common noises.

On the other hand, since in the quantum detector method the vacuum auto-correlation functions are the essential physical quantities, and since according to fluctuation-dissipation theorem(s) (FDT) they are related to the linear (equilibrium) response functions to an initial condition/vacuum, the FDT approach has been of interest for years [20]. Here, I would like to point out the recent generalization of FDT to some classes of out of equilibrium relaxational systems [22] that looks promising for the nonstationary “vacuum baths” as well. At the formal level, the generalization is quite simple by introducing a two-time FD ratio  $X(t, t')$  in the FDT, i.e. one writes  $T_{\text{eff}} R(t, t') = X(t, t') \frac{\partial C}{\partial t'}(t, t')$ , where  $T_{\text{eff}}$  is an effective

temperature,  $R$  is the response function, and  $C$  is the associated correlation function. It is the FD ratio that is employed to perform the separation of scales. There are already interesting results regarding out of equilibrium effective temperatures, which in general are time-scale-dependent quantities [23]. Precisely such kind of quantities are needed for relativistic VFNs [15], which correspond naturally to out of equilibrium conditions.

## 5. Conclusion

The main conclusion of this work is that one may well ascribe radiometric meaning to the curvature invariants of stationary worldlines. This means that these invariants may be used as radiometric quantities beyond the common “thermodynamic” temperature, which is of limited value in the worldline approach. For nonstationary VFNs, the usage of NCT transform and/or considerations of FDT type may be quite useful in the processing of the VFN signals. The experimental challenge is of course the detection of VFN spectra.

## Acknowledgment

This work was partially supported by the CONACyT project 458100-5-25844E.

## Appendix: Getting the SVES formulas

One can calculate all sorts of SVES by means of the following general formula

$$S(E) = \left| \frac{E^2}{4\pi^3} \int_{-\infty}^{\infty} e^{-iEs} ([x_{\mu}(s) - x_{\mu}(0)][x^{\mu}(s) - x^{\mu}(0)])^{-1} ds \right| = \frac{E^2}{4\pi^3} |I|, \quad (A1)$$

where  $x^{\mu}(s)$  is an arbitrary point on the worldline and  $x^{\mu}(0)$  is the initial point. The signature of the Minkowski metric is  $\eta_{\mu\nu} = (1, -1, -1, -1)$ . I confirm Letaw’s results by sketching the calculation of the integral  $I$  for the six stationary cases. Simple details that have been skipped by Letaw can be found here. In general, the VFN spectra present a fourth power of a Frenet-type invariant as a scaling parameter ( $\kappa$ ,  $\rho$  and  $\sigma$ , respectively) that should be taken into account when performing the calibration of the spectra.

### 1. The recta.

$x^{\mu}(s) = (s, 0, 0, 0)$ ;  $x^{\mu}(0) = (0, 0, 0, 0)$ . The integral is

$$I_1 = \int_{-\infty}^{+\infty} \frac{e^{-iEs}}{s^2} ds. \quad (A2)$$

It can be obtained by (i) Cauchy’s residue theorem and (ii)  $e^{-iEs} = (1 - iEs + \dots)$ . The value of the integral is  $\pi i(-iE) = \pi E$ , and therefore one gets the cubic spectrum. This inertial zero-point cubic spectrum will appear in all the other five stationary spectra as an additive background and therefore one may take into account only the non-cubic contributions as a measure of noninertial vacuum effects.

### 2. The hyperbola.

$x^{\mu}(s) = \kappa^{-1}(\sinh \kappa s, \cosh \kappa s, 0, 0)$  and  $x^{\mu}(0) = \kappa^{-1}(0, 1, 0, 0)$ . Now the integral is

$$I_2 = \int_{-\infty}^{+\infty} \frac{e^{-i\epsilon_{\kappa} u}}{2(\cosh u - 1)} du. \quad (A3)$$

Writing  $e^{-i\epsilon_\kappa u} = \cos \epsilon_\kappa u - i \sin \epsilon_\kappa u$ , one makes use of formula 3.983.3 at page 505 in the fourth edition of the *Table* of Gradshteyn and Ryzhik (GR) to get

$$\int_0^{+\infty} \frac{\cos ax}{\cosh x - 1} dx = -(\pi a) \text{cth}(\pi a) , \quad (A4)$$

and of the Cauchy theorem for the integral in the sinus function

$$-i \int_{-\infty}^{+\infty} \frac{\sin(\epsilon_\kappa x) dx}{\cosh x - 1} = \pi \epsilon_\kappa . \quad (A5)$$

Thus

$$I_2 = -\pi \epsilon_\kappa \text{cth}(\pi \epsilon_\kappa) + \frac{\pi \epsilon_\kappa}{2} = \pi \epsilon_\kappa [1 - \text{cth}(\pi \epsilon_\kappa)] - \frac{\pi \epsilon_\kappa}{2} = -2\pi \epsilon_\kappa \frac{1}{e^{2\pi \epsilon_\kappa} - 1} - \frac{\pi \epsilon_\kappa}{2} , \quad (A6)$$

where the first term leads to the Planckian spectrum and the latter to the cubic zero-point contribution.

### 3. The helix.

$x^\mu(s) = \rho^{-2}(\tau \rho s, \kappa \cos \rho s, \kappa \sin \rho s, 0)$ ,  $x^\mu(0) = \rho^{-2}(0, \kappa, 0, 0)$ . The integral reads

$$I_3 = \rho^3 \int_{-\infty}^{+\infty} \frac{e^{-i\epsilon_\rho u}}{2\kappa^2(\cos u - 1) + \tau^2 u^2} du . \quad (A7)$$

According to Letaw this integral is non-analytic and indeed I was not able to find any helpful formula in the GR *Table*.

### 4. The semicubical parabola.

$x^\mu(s) = (s + \frac{1}{6}\kappa^2 s^3, \frac{1}{2}\kappa s^2, \frac{1}{6}\kappa^2 s^3, 0)$ ,  $x^\mu(0) = (0, 0, 0, 0)$ . The integral reads

$$I_4 = \kappa \int_{-\infty}^{+\infty} \frac{e^{-i\epsilon_\kappa u} du}{u^2(1 + \frac{1}{12}u^2)} = \kappa I_1 - \kappa \int_{-\infty}^{+\infty} \frac{e^{-i\epsilon_\kappa u} du}{(12 + u^2)} . \quad (A8)$$

Of interest is only the second integral that can be found in the GR *Table* at page 359

$$\int_{-\infty}^{+\infty} \frac{e^{-ipx} dx}{a^2 + x^2} = \frac{\pi}{|a|} e^{-|ap|} , \quad (A9)$$

for  $a > 0$  and  $p$  real. Thus, one gets

$$\int_{-\infty}^{+\infty} \frac{e^{-i\epsilon_\kappa u} du}{(12 + u^2)} = \frac{\pi}{\sqrt{12}} e^{-\sqrt{12}\epsilon_\kappa} . \quad (A10)$$

The final result is

$$S_4 = \frac{\kappa^4 \epsilon_\kappa^2}{4\pi^2 \sqrt{12}} e^{-\sqrt{12}\epsilon_\kappa} . \quad (A11)$$

### 5. The catenary.

$x^\mu(s) = \sigma^{-2}(\kappa \sinh \sigma s, \kappa \cosh \sigma s, \tau \sigma s, 0)$  and  $x^\mu(0) = \sigma^{-2}(0, \kappa, 0, 0)$ . The integral is of the type

$$I_5 = \int_{-\infty}^{+\infty} \frac{\sigma^3 e^{-i\epsilon_\sigma u} du}{2\kappa^2(\cosh u - 1) - \tau^2 u^2} . \quad (A12)$$

This integral turns into  $I_2$  and  $I_4$  in the limits mentioned in the text, respectively, but again there is no helpful formula in the GR *Table*, and thus  $I_5$  appears to be non-analytic.

## 6. The helicoid (helix of variable pitch).

$x^\mu(s) = (\frac{\Delta}{RR_+} \sinh(R_+s), \frac{\Delta}{RR_+} \cosh(R_+s), \frac{\kappa\tau}{R\Delta R_-} \cos(R_-s), \frac{\kappa\tau}{R\Delta R_-} \sin(R_-s))$  and  $x^\mu(0) = (0, \frac{\Delta}{RR_+}, \frac{\kappa\tau}{R\Delta R_-}, 0)$ , where  $\Delta^2 = \frac{1}{2}(R^2 + \kappa^2 + \tau^2 + \nu^2)$ ;  $R^4 = (\kappa^2 + \tau^2 + \nu^2)^2 - 4\kappa^2\tau^2$ ;  $R_+^2 = \frac{1}{2}(R^2 + \kappa^2 - \tau^2 - \nu^2)$ ;  $R_-^2 = \frac{1}{2}(R^2 - \kappa^2 + \tau^2 + \nu^2)$ ; leading to the following integral

$$I_6 = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{e^{-iEs} ds}{(\frac{\Delta}{RR_+})^2 [\cosh(R_+s) - 1] + (\frac{\kappa\tau}{R\Delta R_-})^2 [\cos(R_-s) - 1]} . \quad (A13)$$

This is the most complicated non-analytic stationary case, with no helpful formula in the GR *Table*.

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